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INFLUENCE OF BUILDING EXISTENCE ON SEISMIC LIQUEFACTION OF SUBSOILS

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SUMMARY

A new simplified method is presented to evaluate seismic liquefaction potential of subsoils of buildings. Based on a review of the limited number of publications so far available, dealing with the related theme, the authors have devised the method in such a way that it is simple to handle and has clear physical insight and sufficient engineering accuracy, by taking advantage of Seed's simplified method for free ground sites and of the cone model concept well developed by Meek and Wolf in dynamics in recent years as well as by some American and German scholars in soil statics early since the 1930s. A simple example showed the reasonableness and tendency in good accord with the results of small model tests and of finite element analyses. Therefore, it would have a broad prospect of engineering application. © 1997 by John Wiley & Sons, Ltd.

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KEY WORDS: seismic liquefaction; building subsoil; cone model; Seed's method; soil–building interaction

INTRODUCTION

As is well known, up to the present soil liquefaction evaluation methods available have been set up largely for the free field without buildings. These methods may be divided into four groups, i.e. (1) Seed's simplified method; (2) seismic response analysis method including both total and effective stress approach; (3) empirical formulae; and (4) probabilistic and statistical method. Since the 1964 Niigata Earthquake and Alaska Earthquake brought about an extensive damage of soil liquefaction many investigators have devoted much attention to study seismic liquefaction problems and made remarkable achievements as shown in the methods of four groups cited above. Among them, we would like to give special respect and memory herein to the late professor H. B. Seed, who had made great pioneering contribution to the subject and devised a framework to solve the problem while keeping the unity of theoretical and practical consideration.

When buildings existed the pattern of liquefaction would be different from that in free ground. However, seismic liquefaction evaluations of subsoils of buildings have not yet been resolved in such a way that general and practicable methods with sufficient accuracy and plausibility can be established for the use of engineering design and analysis. This fact may be due to that (1) building subsoils are much more complicated to deal with in contrast with free field of ground since the existence of building makes a non-uniform stress field and a complicated wave propagation pattern due to soil–structure interaction and (2) a variety of influencing factors relevant to buildings such as size, shape, rigidity, base area, buried depth of foundation, dead and

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seismic loading, etc. adds broad variations of the results and requires separate numerical analyses for each particular case, bringing large time and manpower consumption. In a few literatures concerning this topic we mention briefly the results as follows.

Yoshiaki *et al.*[1] have made shaking table tests, using a sand box with footing model, and, at the same time, finite element analyses of the test arrangement. Their conclusions were that soils directly beneath the footing would be harder to liquefy than free field soils, whereas the region near the external verge of footing was more easy liquefable than free field region.

Liu *et al.*[2] have made also shaking table tests with sand box containing model footings, and obtained results similar to but more detailed than those of Yoshiaki *et al.* They suggested a weak zone of liquefaction resistance locating around the line starting from the external end point of footing base and inclining 45° to the horizontal direction.

Finn *et al.*[3] have made model tests on centrifuge and finite element analysis of the testings by TARA-3 program. They obtained that pore pressure in soils near the external verges be larger than in free field region.

Popescu *et al.*[4] have published their contribution to the VELACS project consisting of centrifuge model tests and numerical simulation by DYNAFLOW. v 93 program. Unfortunately, only three pore pressure transducers located at the central line of the model footing were used in the testing and thus no results of the verge region were picked up, although they concluded that the agreement between test and numerical results were overall good.

Jun-Tsai Hwang *et al.*[5] have conducted a two-dimensional effective stress analysis of seismic liquefaction considering soil–structure interaction and obtained following results, i.e. the zone directly under the structure is less prone to liquefaction while the zone outside the structure more susceptible to liquefaction, as compared with the free field condition. In other words, the zones outside the structure reach liquefaction first, followed by the soils far away from the structure, and the zone under structure liquefy last.

In summary, though there have been only limited results of study on the problem and the small model tests were not sufficient to obtain rigorous quantitative data due to lack of large number of transducer, influence of transducer disturbance and boundary reflection of test container, etc. it can, at least, be seemingly valid, viz. the existence of three zones having different liquefaction potential, high for the outside zone, medium for the free field zone, and low for the zone beneath building, when a building presence on foundation soils.

In what follows, we present a solution method simple to handle and in coincidence with the general trend concluded above. It is of sound physical background and application prospect.

PROPOSED SIMPLIFIED EVALUATION METHOD

Although some relatively rigorous methods, such as the finite element analyses mentioned above and conducted recently by us,[6] may be effective to solve the problem, their disadvantages of much time and labour consumption, of strong dependence on many material parameters which possess large variations and involve many uncertainties for soils, and of the inaccuracy in interpretation of the computational results for each element, etc. make a simplified method simple to handle, clear of physical insight, and having enough engineering accuracy very necessary for engineering application. Especially, it is well known that for soil media which are a loose deposition of granular particles of various size and mineral under a variety of environment to establish a completely valid theory describing even their static behaviours would be very difficult, if not impossible, not to say dynamic behaviors. So to seek simplified methods seemed not only reasonable but also imperative. We thus intend to propose a method that used the Seed's simplified method, which has been broadly applied to evaluation of soil liquefaction in the free field and has long withstood objection and interrogation, to check the liquefaction potential of building subsoil and takes account of the building–soil interaction effect by using the concept of cone model to determine the stresses in the semi-space, as used in soil statics early since the 1930s, and initiated and well demonstrated in soil dynamics by Meek and Wolf in recent years.

Omitting the description of conventional comprehensive methods, we put emphasis on the presentation of

the simplified method in what follows. As commonly doing in structure dynamics problem, we may proceed in two ways, namely given displacement or given stress (or force) and the latter will be shown to be more convenient and appropriate for application and linking the tradition. And therefore we will concentrate more attention to this type of solution.

To show the basic idea and principle of the method proposed by us, the following assumptions are adopted tentatively. For more general conditions some of them may be altered, making thus only some manipulations of more complex but feasible procedure.

1. The building is rigid as a whole.
2. The building is situated on the ground surface and represented only by a footing in what follows.
3. The soils may be regarded as elastic under dead and seismic loading condition.
4. The rocking component of building motion is neglected in evaluation of liquefaction potential because only shear stress is important to induce residual pore water pressure.
5. A plane shearing body wave SH is vertically upward input from the source of earthquake.

Based on the above assumptions and the substructure method to deal with the structure reaction to the seismic input we can sum up the proposed method as consisted of the following procedures, viz.

(I) For given displacement

1. Determine the free field ground motion u_f. If the input motion from the earthquake source is $u_i = u_{in} f(t - z/c)$ then the free field motion of homogeneous elastic media will be

$$u_f = u_{in} f\left(t - \frac{z}{c_s}\right) + u_{in} f\left(t + \frac{z}{c_s}\right) \tag{1}$$

where $f(t)$ is an input function, c_s a shear wave velocity, z a vertical ordinate, positive downward.

2. Determine the additional ground motion u_j induced by the presence of building. From the above assumptions it can be shown that u_j may be determined by the model of a semi-infinite ground under the action of rigid structure's inertial force $M\ddot{u}_f$ (see, for example, Reference 7). Suppose the transitional displacement of the footing base, u_0, has been determined by any method available, including even also the cone models as systematically presented in a new book entitled 'Foundation Analysis Using Simple Physical Models' by J. P. Wolf, published in 1994 (note u_0 is different from u_f ($z = 0$), but the displacement of the footing base relative to the free field motion), then we may determine u_j from u_0 as follows.

Though this type of problem belongs to a classical problem of elastodynamics and has some solutions for certain cases, they often are cumbersome to use and usually they put emphasis only on the response of structure rather than that of subsoils. Therefore, we use a simplified technique well developed by Meek and Wolf[8,9] in recent years, Cone models, to determine the ground motion due to footing vibration inputs. As the reasonableness, simplicity, generality, and accuracy, etc. of the model have been proved and clarified quite fully by them so we will generalize it directly to our problem in what follows, omitting detailed description.

As shown in Figure 1, the horizontal ground motion u_j may be computed by the use of the cone model theory when the footing base motion u_0 is known.

For a circular footing of radius r_0, resting on the ground surface of an isotropic homogeneous semi-space (now a truncated cone), u_j due to the footing base excitation $u_0(t)$ may be written as[8]

$$u_j(z, t) = \frac{z_0}{z + z_0} u_0\left(t - \frac{z}{c_s}\right) \tag{2}$$

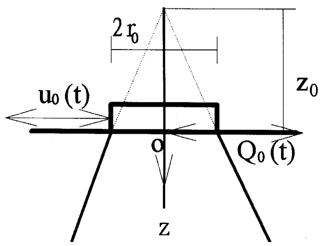

Figure 1. Use of cone model to determine u_j from u_0 for the semi-space

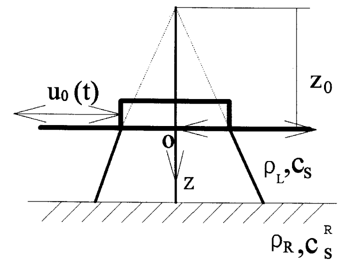

Figure 2. Use of cone model to determine u_j from u_0 for the soil layer

with z measured from the free surface. While the shear stress may be obtained approximately from

$$\tau_j(z,t) = G\frac{\partial u_j(z,t)}{\partial z} = \frac{G}{z_0}\left(\frac{z_0}{z+z_0}\right)^2 u_0\left(t-\frac{z}{c_s}\right) + \frac{G}{c_s}\left(\frac{z_0}{z+z_0}\right)\dot{u}_0\left(t-\frac{z}{c_s}\right) \tag{3}$$

if we consider a condition of standing wave, i.e. $u_0(0)=\overline{u_0}e^{iwt}$, $u_j(z)=\overline{u_j}(z)e^{iwt}$, then we have

$$u_j(z,t) = \frac{z_0}{z+z_0}\,\overline{u_0}e^{iwt} \tag{4}$$

$$\tau_j(z,t) = G\frac{z_0}{(z+z_0)^2}\,\overline{u_0}e^{iwt} \tag{5}$$

For cases of surface soil layer resting on elastic rock half space, Figure 2, we may write, following Wolf:[6]

$$u_j^{L}(z,t) = \frac{z_0}{(z_0+z)}u_0\left(t-\frac{z}{c_s}\right) + \sum_{i=1}^{k}(-\alpha)^i\left\{\frac{z_0u_0\left(t-\dfrac{2id}{c_s}+\dfrac{z}{c_s}\right)}{(z_0+2id-z)} + \frac{z_0u_0\left(t-\dfrac{2id}{c_s}-\dfrac{z}{c_s}\right)}{(z_0+2id+z)}\right\} \tag{6}$$

$$\tau_j^{L}(z,t) = G\frac{\partial u_j(z,t)}{\partial z} = G\sum_{i=1}^{k}(-\alpha)^i\frac{\partial}{\partial z}\left\{\frac{z_0u_0\left(t-\dfrac{2id}{c_s}+\dfrac{z}{c_s}\right)}{(z_0+2id-z)} + \frac{z_0u_0\left(t-\dfrac{2id}{c_s}-\dfrac{z}{c_s}\right)}{(z_0+2id+z)}\right\} + \tau_j(z,t) \tag{7}$$

where $-\alpha$ is the reflection coefficient for displacement wave, which can be approximately determined by

$$-\alpha = -\frac{\rho_R c_s^R - \rho_L c_s}{\rho_R c_s^R + \rho_L c_s} \tag{8}$$

although the rigorous formula should be more complex, which is already a function of complex variable in terms of frequency, layer depth, and two wave resistances even in the frequency domain, see, for example, Men *et al.*[10] and Brekovskikh,[11] not to say in time domain. The meaning of the symbols are as follows:

ρ_R, ρ_L	mass density of the rock and soil, respectively,
c_s^R, c_s^L	shear wave velocity of the rock and soil, respectively,
$u_0(t)$	input translational displacement of footing base to the cone
d	depth of rock surface
k	the number equal to the largest j for which at least one of the arguments of u_0 for a specific z and t is positive

i an integer variable from 1 to k
$\tau_j(z, t)$ the primary term, as determined from equations (2) and (3).

3. Determine the total dynamic shear stress in soils τ_t by

$$\tau_t = \tau_f(z, t) + \tau_j(z, t) \tag{9}$$

where $\tau_f(z, t)$—seismic shear stress in free field, induced by earthquake wave excitation, to be determined from the ground motion described by equation (1) for homogeneous media and somehow more complex equation for layered media not given in detail herein.

After item (3) the procedures will be same as those for case given stress, as given below. So we stop here to avoid repetition.

(II) For given stress

Usually, building's seismic response analyses or seismic design codes define the base shearing force and base torque rather than displacement for easy use in structure design. At the same time, experiences showed that stress evaluation in soils is more reliable and convincible than displacement because soils' discrete, loose and granular nature enable their displacement behaviour to be very complicated especially for large deformation conditions. Therefore, for a long time, since about the beginning of this century, the concept of using likely cone model to evaluate stress distribution in subsoil of foundation has been created and utilized in soil mechanics, Kogler,[12] whereas the settlement evaluation remained so far still by the use of oedometer tests with simple summation of sublayers method. Thus much it seems more reasonable to use stress method in soil dynamic problems also, in our opinion.

1. Determine the free field shear stress τ_f. Although the primary earthquake excitation emitting from the source would be in type of stress or, precisely speaking, unloading stress waves, Men and Cui,[13] it was long customary to use the ground acceleration to describe and measure seismic intensity in the professionals. So we would like to use this well treated technique and clear idea in our solutions.

First, we know the peak acceleration a_{max} in control point of the free ground surface, and may obtain τ_f by making response analysis of the site by means of any suitable technique including even that some more rigorous. At moment, we use the Seed's appoximate method to evaluate τ_f as

$$\tau_f = 0{\cdot}65\gamma z \frac{a_{max}}{g} \tag{10}$$

in which the stress reduction factor γ_d has been neglected to achieve safety and simplicity, and γ is the saturated unit weight of soil, g is the gravity acceleration.

2. Determine the additional shear stress τ_c induced by the presence of building.

Because we concern more about shear stresses here, we can start directly from the cone concept to derive out the formula below (since the base shear $Q_0 = F(t)A$ is given where A = base area)

$$\tau_c(z, t) = \left(\frac{z_0}{z_0 + z}\right)^2 F\left(t - \frac{z}{c_s}\right) \tag{11}$$

for a circular footing resting on an semi-infinite homogenous media, and

$$\tau_c(z, t) = \frac{z_0^2}{(z_0 + z)^2} F\left(t - \frac{z}{c_s}\right) + \sum_{j=1}^{k} (-\alpha)^j \left\{ \frac{z_0^2 F\left(t - \frac{2jd}{c_s} + \frac{z}{c_s}\right)}{(z_0 + 2jd - z)^2} + \frac{z_0^2 F\left(t - \frac{2jd}{c_s} - \frac{z}{c_s}\right)}{(z_0 + 2jd + z)^2} \right\} \tag{12}$$

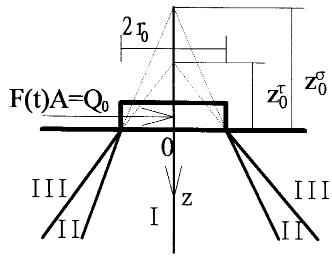

Figure 3. An example showing the use of cone models to determine τ_t from Q_0 for checking liquefaction of subsoil

for cases of surface soil layer resting on elastic rock half space (Figure 2), where $-\alpha$ is the reflection coefficient which can be approximately determined by

$$-\alpha = \frac{\rho_R c_s^R - \rho_L c_s^L}{\rho_R c_s^R + \rho_L c_s^L} \tag{13}$$

Note here equation (13) is valid for stress waves and just the -1 times of equation (8) which is valid for displacement waves, and $F(t)$ is the input shear stress function on base plan of the footing, other parameters are the same as in those in equation (8).

For a strip footing condition we have obtained the additional shear stress as follows:[6]

$$\tau_{ad} = \frac{z_0}{z_0 + z} F\left(t - \frac{z}{c_s}\right) \tag{14}$$

with $F(t)$ now input shear stress function measured in per meter of the footing length.

3. Determine the total dynamic shear stress in soils τ_t, by

$$\tau_t = \tau_f(z, t) + \tau_c(z, t) \tag{15}$$

where $\tau_f(z, t)$ is the seismic shear stress in free field.

4. In a similar way, compute normal stresses in soils to account for the effect of consolidation pressure to test specimen for soil liquefaction resistance testing

$$\sigma_t = \sigma_f + \sigma_{ad}(z, t) \tag{16}$$

where σ_f is the soil effective self-weight pressure, σ_{ad} is the additional static and seismic normal pressure caused by the building.

5. Use triaxial dynamic compression apparatus or dynamic simple shear test devise to obtain the soil liquefaction resistance τ_R for relevant points to be studied.

6. Make comparison between τ_t and τ_R for the every point to see if that soil point may liquefy, as do in Seed's simplified method.

As a complete example we give a concrete solution and discussion of the case as shown in Figure 3.

According to the cone model theory, the influence of structure, now a circular rigid footing, confines by the two cones, one shear and another compression, and the total solution are divided into three zones as named as I, II and III. The stresses in each zone may be computed approximately as follows:

Zone I:

$$\tau_t = \tau_f + \tau_{ad}$$

$$\tau_f = 0{\cdot}65\gamma z \frac{a_{max}}{g}$$

Table I. z_0/r_0

Poisson's ratio v	0	$\frac{1}{4}$	$\frac{1}{3}$	v
Shear cone	0·785	0·687	0·655	$\frac{\pi}{8}(2-v)$
Compres. cone	1·571	1·767	2·094	$\frac{\pi}{4}(1-v)\left(\frac{c}{c_s}\right)^2$ $v \leqslant \frac{1}{3}$ $c = c_\rho$ $\frac{1}{3} < v \leqslant \frac{1}{2}$ $c = 2c_s$

Table II. τ_t/σ_t for each zone

		Zone I			Zone II			Zone III
r_0	z	$W = 10$	$W = 20$	$W = 30$	$W = 10$	$W = 20$	$W = 30$	
$r_0 = 1$ m	0	0·250	0·250	0·250	∞	∞	∞	0·260
	1	0·165	0·138	0·124	0·359	0·460	0·559	0·260
	2	0·209	0·179	0·159	0·279	0·299	0·318	0·260
	3	0·233	0·212	0·196	0·267	0·274	0·280	0·260
	5	0·250	0·242	0·235	0·261	0·263	0·265	0·260
$r_0 = 2$ m	0	0·250	0·250	0·250	∞	∞	∞	0·260
	1	0·219	0·197	0·198	0·311	0·360	0·413	0·260
	2	0·237	0·221	0·220	0·272	0·285	0·297	0·260
	3	0·246	0·236	0·235	0·265	0·270	0·275	0·260
	5	0·255	0·249	0·244	0·261	0·262	0·264	0·260

$$\tau_{ad} = \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\frac{a_{max}}{g} \tag{17}$$

$$\sigma_t = \sigma_f + \sigma_{ad}$$

$$\sigma_f = \gamma' z$$

$$\sigma_{ad} = \left(\frac{z_0^\sigma}{z_0^\sigma + z}\right)^2 \frac{W}{A}\left(1 \pm \frac{a_{max}}{2g}\right) \tag{18}$$

Zone II:

$$\begin{aligned} \tau_t &= 0{\cdot}65\gamma z\frac{a_{max}}{g} + \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\frac{a_{max}}{g} \\ \sigma_t &= \gamma' z \end{aligned} \tag{19}$$

Zone III:

$$\begin{aligned} \tau_t &= \tau_f = 0{\cdot}65\gamma z\frac{a_{max}}{g} \\ \sigma_t &= \gamma' z \end{aligned} \tag{20}$$

where W is the total static load applied on footing base, A the area of footing base, equal to πr_0^2, g the gravity acceleration, γ the saturated unit weight of soil, γ' the effective unit weight for soil, z_0^τ, z_0^σ are the apex height of shear cone and compression cone, respectively, dependent on Poisson's ratio v as given by Wolf[9] as in Table I. Stress reduction factor r_d has been neglected in evaluating τ_f, and the vertical acceleration is taken routinely to be 0·5 times the horizontal acceleration.

It can be seen from the table that z_0^τ is always smaller than z_0^σ so the shear cone is always wider than the compression cone, and therefore, the ratio τ_t/σ_t will be most large in zone II, small in zone I and zone III. As to which one of zone I and zone III has greater ratio τ_t/σ_t, it depends on many factors relevant to building and soil and earthquake intensity, as will be shown later. But generally speaking, τ_t/σ_t in zone I would be a little smaller than that in zone III. Because the liquefaction potential of soil is directly (but not linearly) proportional to τ_t/σ_t so this ratio may be regarded roughly as an indicator of liable liquefaction.

$$\tau_t/\sigma_t = \Phi\left(z, z_0^\tau, z_0^\sigma, \frac{W}{A}, r_0, \frac{a_{\max}}{g}\right). \tag{21}$$

For an example, taking $v = 0{\cdot}3$, $\gamma = 2{\cdot}0\ \mathrm{t/m^3}$, $\gamma' = 1{\cdot}0\ \mathrm{t/m^3}$, $r_0 = 1, 2\ \mathrm{m}$, $W = 10, 20, 30\ \mathrm{t}$, $a_{\max}/g = 0{\cdot}2$, and substituting these data into equations (17)–(21) for each zone we get the results as given in Table II.

It can be seen from Table II that beneath the footing in zone I τ_t/σ_t are smaller than in zone III but the differences are becoming smaller with increasing depth, which agree well with the physical insight that the normal pressure effects diminish as depth increases, W/A decreases, and r_0 increases. The greatest τ_t/σ_t in zone II is due to the combination of largest τ_t and smaller σ_t in the ring cone space. Meanwhile, these results coincide well with, and explain more clearly, the common trend so far shown in the literature as mentioned in the first paragraph.

It can be noted also in zone II τ_t/σ_t has a singular point at $z = 0$, the end point of footing, which can explain from the physical insight. And beyond certain depths, about 5 m for the example's case of $W = 10$, τ_t/σ_t tends to equal that of the free field, in other words the influence of the building is likely disappeared.

CONCLUSIONS

Taking account of building–soil interaction effect and Meek and Wolf's cone model theory we proposed a simplified method of seismic liquefaction evaluation for building's subsoils, which is simple to handle, needs not advanced mathematical and numerical skill, has clear physical idea and tolerable engineering accuracy. Therefore, it seems that the method may be used widely for ordinary building subsoils to check liquefaction potentials during earthquake or under machine vibration. For important buildings a larger safety coefficient should be used at present stage before a further refinement and improvement will be made.

As a verification we made also two-dimensional effective stress finite element analyses of the same problem and obtained trends in good agreement with those of the simplified method. The detailed statement has been partly published elsewhere.[6]

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